

Blizzard Bag Day #2

End Behavior, zeros, and turning points review

Complete the **ALL OF THE PROBLEMS** on the the following pages.

For refreshers on how to do the material, look back to Notes for section 2-2

Many of these concepts will be on your final exam so it is important to review.

2-2 Study Guide and Intervention

Polynomial Functions

Graph Polynomial Functions A polynomial function of degree n is a function of the form

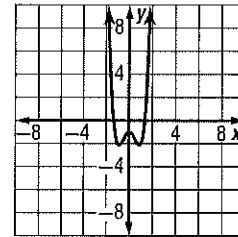
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0,$$

where n is a nonnegative integer and $a_0, a_1, a_2, \dots, a_{n-1}, a_n$ are real numbers with $a_n \neq 0$. Maxima and minima are located at **turning points**. A polynomial function of degree n has at most n distinct real zeros and at most $n - 1$ turning points.

Example 1 Graph $f(x) = 2x^4 - 3x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.

The degree is 4 and the leading coefficient is 2. Because the degree is even and the leading coefficient is positive,

$$\lim_{x \rightarrow -\infty} f(x) = \infty \text{ and } \lim_{x \rightarrow \infty} f(x) = \infty.$$



Example 2 State the number of possible real zeros and turning points of $f(x) = x^3 + 2x^2 - x - 2$. Then determine all of the real zeros by factoring.

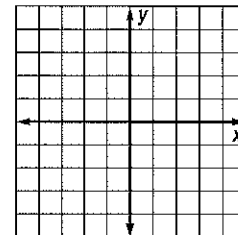
The degree is 3, so f has at most 3 distinct real zeros and at most $3 - 1$ or 2 turning points. To find the real zeros, solve the related equation $f(x) = 0$ by factoring.

$x^3 + 2x^2 - x - 2 = 0$	Set $f(x)$ equal to 0.
$x^2(x + 2) - 1(x + 2) = 0$	Group the terms and find the GCF.
$(x^2 - 1)(x + 2) = 0$	Regroup using the Distributive Property.
$(x + 1)(x - 1)(x + 2) = 0$	Factor completely.

So, f has three distinct real zeros, $x = -1$, $x = 1$, and $x = -2$. The graph has two turning points.

Exercises

- Graph the function $f(x) = -3x^5 + 2x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.



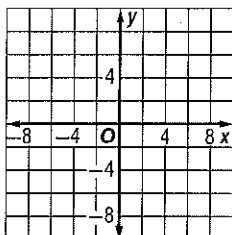
- State the number of possible real zeros and turning points of $f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$. Then determine all of the real zeros by factoring.

2-2 Practice

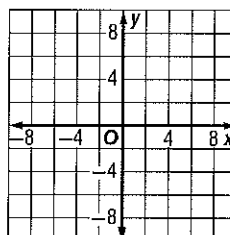
Polynomial Functions

Graph each function. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.

1. $f(x) = -7x^5 - 2x^3 + 3x^2 + 5$



2. $f(x) = x^6 - 2x^4 + 1$



State the number of possible real zeros and turning points of each function. Then determine all of the real zeros by factoring.

3. $f(x) = x^3 - 3x^2 - x + 3$

4. $f(x) = x^4 - 5x^2 - 36$

5. The amount of food energy produced by farms increases as more energy is expended. The following table shows the amount of energy produced and the amount of energy expended to produce the food.

Energy Input (Calories)	606	970	1121	1227	1318	1455	1636	2030	2182	2242
Energy Output (Calories)	133	144	148	157	171	175	187	193	198	198

- Write a polynomial function to model the set of data.
 - Predict the energy output when the energy input is 800 Calories.
6. For $f(x) = -x(2x + 5)^2(x - 3)$, (a) apply the leading term test, (b) determine the zeros and state the multiplicity of any repeated zeros, (c) find a few additional points, and then (d) graph the function.